Graviton-photon conversion on spin 0 and 1/2 particles

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Abstract

The differential cross-sections for scattering of gravitons into photons on bosons and fermions are calculated in linearized quantum gravity. They are found to be strongly peaked in the forward direction and become constant at high energies. Numerically, they are very small as expected for such gravitational interactions.

1 Introduction

Inspired by string theories, it has recently been suggested that spacetime might have large, extra dimensions so that gravity can be unified with the other interactions at experimentally accessable energies[1][2]. As a consequence, the graviton will then have stronger interactions than in more conventional theories. In addition there will be a large number of Kaluza-Klein excitations of the graviton which also can be important in future experiments[3][4][5][6][7]. These will have couplings of the same kind as the ordinary, massless graviton. For these reasons it will now be of some interest to study the interactions of gravitons with other particles in order to learn more about what to expect within such theoretical frameworks.

We will here calculate the differential cross-section for the graviproduction process $g+p \to \gamma+p'$ where a photon γ is produced in the final state by an incoming graviton g interacting with a matter particle p. Using linearized quantum gravity and the requirements of gauge invariance, the production amplitude is uniquely defined in lowest and dominant order of perturbation theory. In the next section we give a short summary of linearized gravity and the Feynman rules for graviton interactions. In Section 3 we calculate the production cross-section on scalar particles and on Dirac particles in the following section. Finally, there is a short discussion of our results.

2 Linearized theory of gravity

In standard Einstein gravity, the metric $g_{\mu\nu}$ is determined by the energy-momentum tensor $T_{\mu\nu}$ as the solution of the fundamental equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \tag{1}$$

where $R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}$ is the Ricci tensor and $R = R^{\mu}{}_{\mu}$. The constant multiplying $T_{\mu\nu}$ is the squared inverse Planck mass which we for convenience will denote by $f^2 = 8\pi G$. For small fluctuations around the Minkowski metric $\eta_{\mu\nu}$, the full metric is now usually written as[8][9].

$$g_{\mu\nu} = \eta_{\mu\nu} + 2f h_{\mu\nu} \tag{2}$$

where $h_{\mu\nu}(x)$ is the graviton field. When it is small in amplitude, we then have to lowest order

$$R_{\mu\nu} = -f(\Box\, h_{\mu\nu} - \partial_\mu h_\nu - \partial_\nu h_\mu) \tag{3}$$

where $h_\mu = \partial_\lambda h^{\lambda}{}_{\mu} - \frac{1}{2}\,\partial_\mu h^{\lambda}{}_{\lambda}$. It is now easy to verify that this curvature tensor is invariant under the local gauge transformation

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \chi_\nu + \partial_\nu \chi_\mu \tag{4}$$

where $\chi_\mu = \chi_\mu(x)$ is an arbitrary vector function. These gauge transformations correspond to local coordinate transformations.

This local invariance allows us to choose a convenient gauge for the graviton field as one also does in the description of the photon field. The simplest and most common gauge choice is the Hilbert or harmonic gauge defined by the condition $h_\mu = 0$. It is convenient to introduce the barred field

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^{\lambda}{}_{\lambda} \tag{5}$$

which is an idempotent operation, i.e. $\bar{\bar{h}}_{\mu\nu} = h_{\mu\nu}$. The Hilbert gauge condition can then be written as $\partial_\mu \bar{h}^{\mu\nu} = 0$. The Ricci tensor (3) now takes the simple form $R_{\mu\nu} = -f\Box\, h_{\mu\nu}$ and the Einstein equation (1) simplifies to

$$\Box\, \bar{h}_{\mu\nu} = -f T_{\mu\nu} \tag{6}$$

We have thus arrived at the wave equation for the graviton field produced by the source on the right-hand side. Its stucture is the same as for the electromagnetic wave equation in the Lorentz gauge where the source is the electric four-current.

Using the analogy with electromagnetic theory, we can now couple the above graviton field to another matter distribution described by the energy-momentum tensor $T'_{\mu\nu}$ via the interaction

$$\mathcal{L}_{int} = -f h^{\mu\nu} T'_{\mu\nu} \tag{7}$$

This gives a direct coupling between these two matter distributions which is

$$\mathcal{L}_{int} = f^2 \, T'_{\mu\nu} \frac{1}{\Box} \, \bar{T}^{\mu\nu} \tag{8}$$

where the barred energy-momentum tensor $\bar{T}_{\mu\nu}$ is defined as in (5). It can be unbarred with the help of the projection operator

$$P_{\mu\nu\rho\sigma} = \frac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}) \tag{9}$$

to give

$$\mathcal{L}_{int} = f^2 \, T'^{\mu\nu} \frac{P_{\mu\nu\rho\sigma}}{\Box} \, T^{\rho\sigma} \tag{10}$$

In the quantized theory this interaction is due to the exchange of a graviton. We thus have for the graviton propagator in momentum space

$$D_{\mu\nu\rho\sigma}(k) = -\frac{P_{\mu\nu\rho\sigma}}{k^2 + i\epsilon} \tag{11}$$

From Eq. (7) we see that it couples to the energy-momentum tensor with strength f. The situation is very similar to QED where the photon field A_μ couples to the electrical current with strength e which is the electric charge of the particle.

2.1 Graviton polarizations

The description of a free graviton is not completely fixed in the Hilbert gauge. It still allows gauge transformations of the type (4) as long as $\Box\chi_\mu = 0$. One can then impose further conditions on the field tensor which now can be taken to be traceless, ${h^\sigma}_\sigma = 0$. Four more degrees of freedom are removed and one is working in the transverse traceless (TT) gauge[10]. The number of independent field components is then $10 - 4 - 4 = 2$ corresponding to the two helicity states $\oplus$ and $\otimes$ of the graviton. As polarization basis states it is then convenient to take

$$\epsilon^{\mu\nu}(\oplus) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \epsilon^{\mu\nu}(\otimes) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{12}$$

for a graviton with four-momentum $q^\mu = (\omega_{\mathbf{q}}, \mathbf{q})$ and energy $\omega_{\mathbf{q}} = |\mathbf{q}|$ where the three-momentum $\mathbf{q}$ is along the z-axis. We see that the TT gauge conditions $q_\mu \epsilon^{\mu\nu} = \epsilon^\mu{}_\mu = 0$ are satisfied. With this choice one can then do polarization sums based on the projection operator[8]

$$\begin{aligned}\sum_{\lambda=\oplus,\otimes} \epsilon_{ij}(\lambda)\epsilon^\star_{lm}(\lambda) &= \frac{1}{2}\Big[(\delta_{il}\delta_{jm} + \delta_{im}\delta_{jl} - \delta_{ij}\delta_{lm}) - \frac{\delta_{il}q_jq_m + \delta_{im}q_jq_l - \delta_{ij}q_lq_m}{\mathbf{q}^2} \\ &- \frac{\delta_{jm}q_iq_l + \delta_{jl}q_iq_m - \delta_{lm}q_iq_j}{\mathbf{q}^2} + \frac{q_iq_jq_lq_m}{\mathbf{q}^4}\Big] \end{aligned} \tag{13}$$

An alternative method is obtained by making use of gauge invariance. The summation over the two physical helicity states can then be extended to include also the non-physical states which by themselves mutually cancel. Using completeness of all these polarization states, we then have instead

$$\sum_{\lambda=\oplus,\otimes} \epsilon_{\mu\nu}(\lambda)\epsilon^\star_{\rho\sigma}(\lambda) = P_{\mu\nu\rho\sigma} \tag{14}$$

which is the projection operator introduced in (9). In many instances this allows a simpler calculation.

3 Transition vertex elements

Lowest order matrix transition elements which describe the coupling of gravitons to other fields, are well known and can be found in the literature[8][9]. But since we find a few minor, but crucial discrepancies in these formulas, we will present corrected expressions here.

3.1 Coupling to bosons

Let us first consider the coupling of a charged boson with mass m described by the complex Klein-Gordon field $\phi(x)$ to the electromagnetic field $A_\mu(x)$. Gauge invariance gives rise to the conserved electric current $J_\mu = i(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*)$. The coupling of a photon with four-momentum k^μ and polarization vector ϵ^μ is then given by the matrix element

$$\epsilon^\mu\langle p'|J_\mu(0)|p\rangle = \epsilon^\mu(p'_\mu + p_\mu) \tag{15}$$

where p_μ and $p'_\mu = p_\mu + k_\mu$ are the initial and final momenta of the particle. Choosing to work in the Lorentz gauge, we now find invariance of the matrix element under the gauge transformation $\epsilon^\mu \to \epsilon^\mu + ak^\mu$ where a is some arbitrary constant.

The analogous result for the coupling of a graviton with four-momentum q^μ and polarization tensor $\epsilon^{\mu\nu}$ is now seen from (7) to be given by the corresponding matrix element $\langle p'|T_{\mu\nu}(0)|p\rangle$ where $T_{\mu\nu}$ is the energy-momentum tensor. For the boson field under consideration it has the form

$$T^B_{\mu\nu} = \partial_\mu\phi^*\partial_\nu\phi + \partial_\mu\phi\partial_\nu\phi^* - \eta_{\mu\nu}(\partial_\sigma\phi^*\partial^\sigma\phi - m^2\phi^*\phi) \tag{16}$$

Taking the matrix element between the same states, we find the gravitational

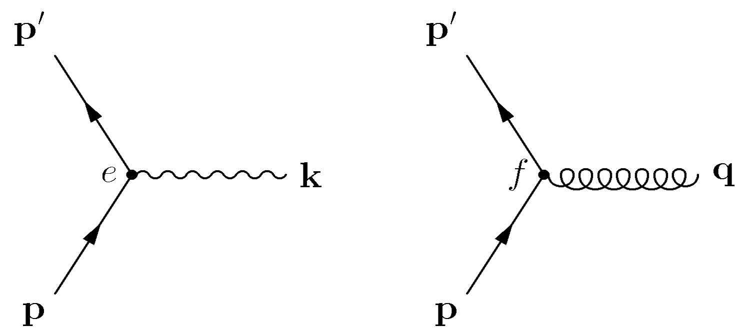

Figure 1: Photon and graviton coupling to particle.

coupling to a boson to be

$$\epsilon^{\mu\nu}\langle p'|T^B_{\mu\nu}(0)|p\rangle = \epsilon^{\mu\nu}[p_\mu p'_\nu + p'_\mu p_\nu - \eta_{\mu\nu}(p\cdot p' - m^2)] \tag{17}$$

Gauge invariance is now verified by noting that under the substitution $\epsilon^{\mu\nu} \to \epsilon^{\mu\nu} + q^\mu\chi^\nu + q^\nu\chi^\mu$ where χ_μ is an arbitrary vector satisfying $q^\mu\chi_\mu = 0$, the coupling remains the same. These two couplings are shown in Fig.1.

There is also a contact term coupling a graviton directly to a photon in the presence of the particle field. This follows from electromagnetic gauge invariance which implies that the partial derivatives $\partial_\mu\phi$ in the energy-momentum tensor (16) must be replaced by the covariant derivatives $(\partial_\mu + ieA_\mu)\phi$. This generates new terms

$$\begin{aligned} T^{AB}_{\mu\nu} = & - \; ie[(\phi^*\partial_\nu\phi - \phi\partial_\nu\phi^*)A_\mu + (\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*)A_\nu \\ & - \; \eta_{\mu\nu}(\phi^*\partial_\lambda\phi - \phi\partial_\lambda\phi^*)A^\lambda] + \mathcal{O}(e^2) \end{aligned} \tag{18}$$

Taking the matrix element between an initial state containing a boson with momentum p together with a photon with momentum k and polarization ϵ^λ and a final state boson with momentum p', we then find the matrix element

$$\langle p'|T^{AB}_{\mu\nu}(0)|p;k,\lambda\rangle = -e[\eta_{\lambda\nu}(p_\mu + p'_\mu) + \eta_{\lambda\mu}(p_\nu + p'_\nu) - \eta_{\mu\nu}(p_\lambda + p'_\lambda)]\epsilon^\lambda \tag{19}$$

This vertex is not gauge invariant in itself, but must be combined with the previous

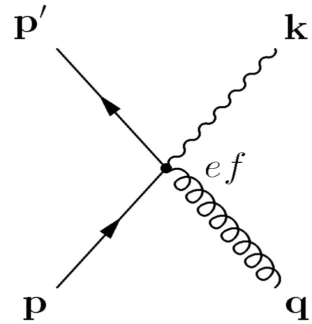

Figure 2: Graviton-photon contact interaction with particle.

couplings to give an overall gauge invariant result. It is illustrated in Fig.2.

Gravitons couple to all kinds of matter and thus also to photons. From the electromagnetic energy-momentum tensor

$$T_{\mu\nu}^{EM} = F_{\mu}{}^{\sigma} F_{\sigma\nu} + \frac{1}{4}\eta_{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} \tag{20}$$

we can now find the matrix element for the transition between a photon with initial momentum k and polarization ϵ^ρ to a final momentum k' and polarization ϵ^σ. We find the result

$$\begin{aligned} \langle k', \sigma | T_{\mu\nu}^{EM}(0) | k, \rho \rangle &= \epsilon^\sigma [k'_\rho (k_\mu \eta_{\sigma\nu} + k_\nu \eta_{\sigma\mu}) + k_\sigma (k'_\mu \eta_{\rho\nu} + k'_\nu \eta_{\rho\mu}) \\ &- \eta_{\sigma\rho}(k'_\mu k_\nu + k'_\nu k_\mu) + \eta_{\mu\nu}(\eta_{\sigma\rho}(k' \cdot k) - k'_\rho k_\sigma) \\ &- (k' \cdot k)(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})]\epsilon^\rho \end{aligned} \tag{21}$$

which is in agreement with the literature[8] except for an overall factor of 1/2. As the previous coupling of a graviton to a boson was shown in Fig.1, this new photon-graviton vertex is correspondingly illustrated in Fig.3.

3.2 Coupling to fermions

The electromagnetic coupling of a photon to a Dirac particle described by the free Lagrangian $\mathcal{L}_F = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$ is via the conserved current $J^\mu = \bar{\psi}\gamma^\mu\psi$. The matrix element for such a transition from an initial fermion state described by the Dirac spinor $u(p)$ where p is the particle four-momentum, to a final fermion state $u(p')$, is then the well-known result $\langle p' | J^\mu(0) | p \rangle = \bar{u}(p')\gamma^\mu u(p)$.

In order to find the energy-momentum tensor for a Dirac particle, we must make the above Lagrangian symmetric in ingoing and outgoing fields. It then takes the

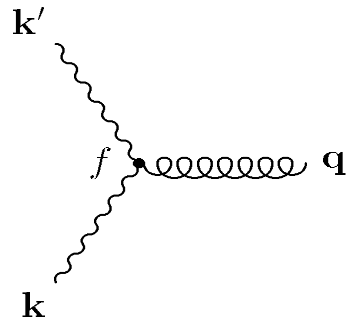

Figure 3: Graviton coupling to photon.

form

$$\mathcal{L}_F = \frac{1}{2}\bar{\psi}i\gamma^\mu(\partial_\mu\psi) - \frac{1}{2}(\partial_\mu\bar{\psi})i\gamma^\mu\psi - m\bar{\psi}\psi \tag{22}$$

The energy-momentum tensor follows by standard methods. It allows the calculation of the matrix element for a transition from an initial fermion state with four-momentum p described by the Dirac spinor $u(p)$, to a final fermion state $u(p')$. We then obtain

$$\langle p'|T^F_{\mu\nu}(0)|p\rangle = \frac{1}{4}\bar{u}(p')[(p_\mu + p'_\mu)\gamma_\nu + (p_\nu + p'_\nu)\gamma_\mu - 2\eta_{\mu\nu}(\not{p} + \not{p}' - 2m)]u(p) \tag{23}$$

where we use the standard notation $\not{p} \equiv \gamma_\mu p^\mu$. It has the same form as for the coupling of a graviton to a boson in Fig.1. We notice that the last term in this fermionic vertex is zero when the initial and final particles are on-shell. This explains why it has been left out in the literature[8].

There is also a graviton-photon contact term here. As for bosons, it can be found from the minimal substitution $\partial_\mu \to \partial_\mu + ieA_\mu$ in the energy-momentum tensor. The resulting four-particle vertex is then

$$\langle p'|T^{AF}_{\mu\nu}(0)|p;k,\lambda\rangle = -\frac{e}{2}\bar{u}(p')[\eta_{\mu\lambda}\gamma_\nu + \eta_{\nu\lambda}\gamma_\mu - 2\eta_{\mu\nu}\gamma_\lambda]u(p)\epsilon^\lambda \tag{24}$$

in analogy with (19) for bosons. It can also be illustrated as in Fig.2.

4 Differential cross-sections

We will now calculate the lowest order scattering amplitude for the inelastic process $g(q) + p \to \gamma(k) + p'$ where a graviton with momentum q scatters on a particle with

momentum p such that it is converted into a photon with momentum k and a final particle with momentum p'. The matter particle has the mass m. Since both the graviton and photon are massless, the kinematics will be the same as for Compton scattering. In the laboratory system where the initial particle is at rest and the incoming graviton has energy ω, the final state photon will come out at an angle θ and with energy ω' given by the Compton formula

$$\frac{1}{\omega'} = \frac{1}{\omega} + \frac{1}{m}(1 - \cos\theta) \tag{25}$$

We will assume that all particles are unpolarized.

From the previous couplings, we find four diagrams which contribute to the process in lowest order. In addition to the two ordinary contributions shown in Fig.4, there

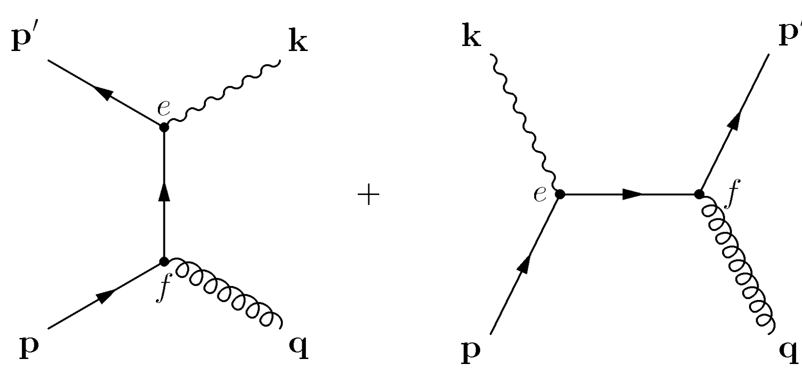

Figure 4: Direct and exchange production of photon by incoming graviton.

is the contact diagram in Fig.2 and also the diagram in Fig.5 where the incoming graviton couples to an exchanged photon.

4.1 Bosonic conversion

When the graviton-photon conversion takes place on a boson, the first diagram in Fig.4 gives rise to the transition matrix element

$$M_{fi}^B(1) = -\frac{ef}{2p\cdot q}\epsilon^{\mu\nu}[2p_\mu p_\nu + p_\mu q_\nu + p_\nu q_\mu - \eta_{\mu\nu}(q\cdot p)](2p'_\lambda + k_\lambda)\epsilon^\lambda \tag{26}$$

The contribution from the second diagram can similarly be written as

$$M_{fi}^B(2) = \frac{ef}{2p\cdot k}\epsilon^{\mu\nu}[2p'_\mu p'_\nu - p'_\mu q_\nu - p'_\nu q_\mu + \eta_{\mu\nu}(q\cdot p')](2p_\lambda - k_\lambda)\epsilon^\lambda \tag{27}$$

There is no internal propagator in the contact term diagram Fig.2 and its contribution to the scattering amplitude can be read off directly from the bosonic vertex (19) and leads to

$$M^B_{fi}(3) = ef\epsilon^{\mu\nu}[\eta_{\lambda\nu}(p_\mu + p'_\mu) + \eta_{\lambda\mu}(p_\nu + p'_\nu) - \eta_{\mu\nu}(p_\lambda + p'_\lambda)]\epsilon^\lambda \tag{28}$$

In the final diagram there is a virtual photon exchanged between the incoming

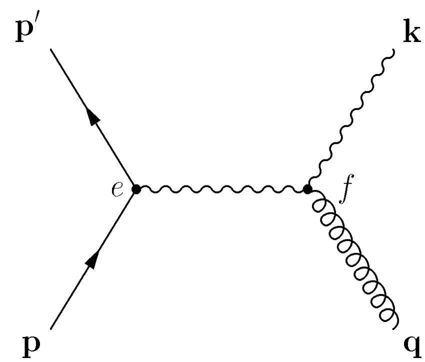

Figure 5: Virtual photon becomes real in the interaction with incoming graviton.

graviton and the outgoing photon. With the corresponding vertex from (21), we find the amplitude

$$\begin{aligned} M^B_{fi}(4) &= -\frac{ef}{2k\cdot q}\epsilon^{\mu\nu}(p'^\rho + p^\rho)[(k_\mu - q_\mu)(k_\rho\eta_{\lambda\nu} - k_\nu\eta_{\rho\lambda}) \\ &+ (k_\nu - q_\nu)(k_\rho\eta_{\lambda\mu} - k_\mu\eta_{\rho\lambda}) + (k_\lambda - q_\lambda)(k_\mu\eta_{\nu\rho} + k_\nu\eta_{\mu\rho} - k_\rho\eta_{\mu\nu}) \\ &+ (k\cdot q)(\eta_{\mu\rho}\eta_{\nu\lambda} + \eta_{\nu\rho}\eta_{\mu\lambda} - \eta_{\mu\nu}\eta_{\rho\lambda})]\epsilon^\lambda \end{aligned} \tag{29}$$

The full scattering amplitude is now

$$M^B_{fi} = M^B_{fi}(1) + M^B_{fi}(2) + M^B_{fi}(3) + M^B_{fi}(4) \tag{30}$$

We check it for electromagnetic gauge invariance in the Lorentz gauge where $\epsilon^\lambda k_\lambda = 0$ by verifying that it is invariant under the gauge transformation $\epsilon^\lambda \to \epsilon^\lambda + ak^\lambda$ where a is an arbitrary constant. Similarly, we verify that it is invariant under the gravitational gauge transformation $\epsilon^{\mu\nu} \to \epsilon^{\mu\nu} + q^\mu\chi^\nu + q^\nu\chi^\mu$ where χ_μ is an arbitrary vector satisfying $q^\mu\chi_\mu = 0$ in the TT gauge where $\epsilon^{\mu\nu}q_\mu = 0$ in momentum space[11].

The differential cross-section for the process $g + p \to \gamma + p'$ in the laboratory system is now given by

$$\frac{d\sigma}{d\Omega}\Big|_{lab} = \frac{1}{64\pi^2 m^2}\Big(\frac{\omega'}{\omega}\Big)^2|M_{fi}|^2 \tag{31}$$

when averaged over initial and summed over final spins. Averaging over the two initial graviton helicities can be done either directly from the TT polarization tensors (12) or using the corresponding polarization projection operator (13). Either way gives the same result

$$|M_{fi}|^2 = e^2 f^2 m^2 (1 + \cos^2\theta)\cot^2\frac{\theta}{2} \tag{32}$$

after also summing over the two final photon helicities[11]. It gives the cross-section

$$\frac{d\sigma}{d\Omega}\Big|_{lab} = \frac{\alpha G}{2}\Big(\frac{\omega'}{\omega}\Big)^2 (1 + \cos^2\theta)\cot^2\frac{\theta}{2} \tag{33}$$

where $\alpha = e^2/4\pi$ is the fine-structure constant and $G = f^2/8\pi$ is Newton's gravitational constant. The factor $(1 + \cos^2\theta)$ is the same as for photon Compton scattering on a scalar particle, while the extra factor $\cot^2(\theta/2)$ makes the differential cross-section diverge in the forward direction. This is due to the exchange of the massless photon in Fig.5. Since the masses of the particles in the initial and final states are the same, this will also be the cross-section for the inverse process $\gamma + p \to g + p'$ which is photoproduction of gravitons.

As an independent check of this surprisingly simple result, we have also evaluated the averaged square matrix element in an arbitrary frame using the covariant projection operator (14) for the graviton and the corresponding result $\sum_\epsilon \epsilon_\mu\epsilon_\nu = -\eta_{\mu\nu}$ for the photon. Introducing the Mandelstam variables $s = (q+p)^2, t = (k-q)^2$ and $u = (p'-q)^2$ with $s + u + t = 2m^2$, we then find

$$\begin{aligned} |M_{fi}|^2 &= e^2 f^2 \Bigg[\frac{(u-m^2)(s-m^2)}{t} + m^2\Big(2 + \frac{(s-m^2)^2}{(u-m^2)^2} \\ &+ 2t\frac{(s+m^2)}{(u-m^2)^2} + t^2\frac{(s+m^2)^2 - 2m^4}{(u-m^2)^2(s-m^2)^2}\Big)\Bigg] \end{aligned} \tag{34}$$

When this is now evaluated in the laboratory system where

$$\begin{aligned} s &= m^2 + 2m\omega \\ t &= -2\omega\omega'(1 - \cos\theta) \\ u &= m^2 - 2m\omega' \end{aligned} \tag{35}$$

we find that (34) simplifies to the previous result (32).

The covariant result (34) also allows us to find the cross-section in the center-of-momentum frame. In the high-energy limit where we can neglect the mass m, it becomes

$$\frac{d\sigma}{d\Omega}\Big|_{CM} = \frac{\alpha G}{2}\cot^2\frac{\theta_{CM}}{2} \tag{36}$$

and shows the same forward peaking. It is independent of energy since it is due to the spin-1 photon exchange that dominates at high energies.

4.2 Fermionic conversion

The Feynman diagrams that contribute to the process when the target particle is a fermion, are the same as in the previous bosonic case. We thus have again four partial amplitudes which we label correspondingly. From the previously derived vertices, they are found to be

$$\begin{aligned}
M^F_{fi}(1) &= -\frac{ef}{8p\cdot q}\epsilon^{\mu\nu}\bar{u}(p')\gamma_\lambda(\not{p}+\not{q}+m)[(2p_\mu+q_\mu)\gamma_\nu \\
&+ (2p_\nu+q_\nu)\gamma_\mu - 2\eta_{\mu\nu}(2\not{p}+\not{q}-2m)]u(p)\epsilon^\lambda \qquad (37)\\
M^F_{fi}(2) &= \frac{ef}{8p\cdot k}\epsilon^{\mu\nu}\bar{u}(p')[(2p'_\mu-q_\mu)\gamma_\nu+(2p'_\nu-q_\nu)\gamma_\mu \\
&- 2\eta_{\mu\nu}(2\not{p}'-\not{q}-2m)](\not{p}-\not{k}+m)\gamma_\lambda u(p)\epsilon^\lambda \qquad (38)\\
M^F_{fi}(3) &= \frac{ef}{2}\epsilon^{\mu\nu}\bar{u}(p')[\eta_{\lambda\mu}\gamma_\nu+\eta_{\lambda\nu}\gamma_\mu-2\eta_{\mu\nu}\gamma_\lambda]u(p)\epsilon^\lambda \qquad (39)\\
M^F_{fi}(4) &= -\frac{ef}{2k\cdot q}\epsilon^{\mu\nu}\bar{u}(p')\gamma^\rho u(p,s)[(k_\mu-q_\mu)(k_\rho\eta_{\lambda\nu}-k_\nu\eta_{\rho\lambda}) \\
&+ (k_\nu-q_\nu)(k_\rho\eta_{\lambda\mu}-k_\mu\eta_{\rho\lambda})+(k_\lambda-q_\lambda)(k_\mu\eta_{\nu\rho}+k_\nu\eta_{\mu\rho}-k_\rho\eta_{\mu\nu}) \\
&+ (k\cdot q)(\eta_{\mu\rho}\eta_{\nu\lambda}+\eta_{\nu\rho}\eta_{\mu\lambda}-\eta_{\mu\nu}\eta_{\rho\lambda})]\epsilon^\lambda \qquad (40)
\end{aligned}$$

The full scattering amplitude $M^F_{fi} = M^F_{fi}(1)+M^F_{fi}(2)+M^F_{fi}(3)+M^F_{fi}(4)$ is again found to be invariant both under electromagnetic and gravitational gauge transformations. Summing over final and averaging over initial spins in the squared amplitude, we find

$$\begin{aligned}
|M_{fi}|^2 &= -e^2f^2\Bigg[2s+t+2\frac{s^2}{t}+m^2\Big(\frac{t}{s-m^2}-\frac{2s}{t} \\
&+ \frac{t(5s+3m^2-t)}{(u-m^2)^2}-\frac{2(s-m^2)^3}{t(u-m^2)^2}+\frac{4s^2t^2}{(s-m^2)^2(u-m^2)^2}\Big)\Bigg] \qquad (41)
\end{aligned}$$

In the laboratory system where the Mandelstam variables take the values (35), we then obtain the cross-section

$$\frac{d\sigma}{d\Omega}\Big|_{lab} = \frac{\alpha G}{2}\Big(\frac{\omega'}{\omega}\Big)^2\left[\frac{\omega\omega'}{m^2}\sin^2\theta+(1+\cos^2\theta)\cot^2\frac{\theta}{2}\right] \qquad (42)$$

We see that the last term is the same as for scattering on a boson while the first term is due to the spin of the target particle.

In the center-of-momentum system the differential cross-section follows directly from the above squared amplitude. At sufficiently high energies we can ignore the mass

m of the target particle and then obtain

$$\frac{d\sigma}{d\Omega}\Big|_{CM} = \alpha G\left[\frac{1}{\sin^2(\theta_{CM}/2)} - \frac{1}{2}\Big(1+\cos^2\frac{\theta_{CM}}{2}\Big)\right] \tag{43}$$

In this limit we thus find again a cross-section which is independent of the energy and strongly peaked in the forward direction.

5 Concluding comments

The typical size of these cross-sections is set by Newton's gravitational coupling constant which in our units is $G = 2.61\times 10^{-66}\,\mathrm{cm}^2$. Graviton-induced creation of photons or photoproduction of gravitons will therefore be completely negligible under ordinary conditions. The only circumstances which could make these reactions physically relevant, would be when the flux of incoming particles became extremely high so to compensate for the very small coupling constant. A directed pulse of gravitons could then be converted into a highly collimated beam of photons. It is not impossible to imagine such a process taking place in the environment of highly compact and strongly relativistic astrophysical objects.

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